Hong Kong Mathematics Olympiad (1991 – 92)
Heat Event (Group)
香港数学竞赛 (1991 – 92)
初赛项目 (团体)

1. A, B, C are three men in a team. The age of A is greater than the sum of the ages of B and C by 16. The square of the age of A is greater than the square of the sum of the ages of B and C by 1632. Find the sum of the ages of A, B and C.
有甲、乙、丙三人，甲的年龄较乙和丙的年龄之和大 16，甲年龄的平方较乙和丙的年龄之和的平方大 1632，求甲、乙、丙的年龄之和。

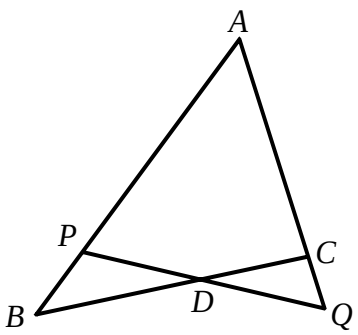
2. a, b, c are non-zero real numbers such that $\frac{a+b-c}{c}=\frac{a-b+c}{b}=\frac{-a+b+c}{a}$.
If $x=\frac{(a+b)(b+c)(c+a)}{abc}$ and $x<0$, find the value of x.
a、b、c 为非零实数，且 $\frac{a+b-c}{c}=\frac{a-b+c}{b}=\frac{-a+b+c}{a}$。
若 $x=\frac{(a+b)(b+c)(c+a)}{abc}$ 及 x 为负数，求 x 的值。

3. An interior angle of an n-sided convex polygon is $x°$. The sum of the other interior angles is $2468°$. Find x.
一凸 n 边形的一个内角是 $x°$，其他各内角的和是 $2468°$，求 x。

4. When a positive integer N is divided by 4, 7, 9, the remainders are 3, 2, 2 respectively. Find the least value of N.
当正整数 N 除以 4、7、9 时，其余数依次为 3、2、2。求 N 的最小值。

5. Find the remainder when 10^{1991} is divided by 7.
求 10^{1991} 除以 7 的余数。

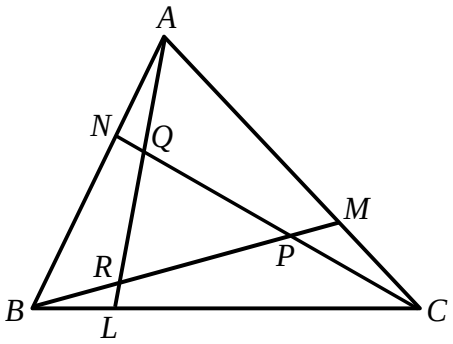
6. In figure 1, $BD=DC$, $AP=AQ$. If $AB=13$ cm, $AC=7$ cm and $AP=x$ cm, find x.
在图 1 中，$BD=DC$、$AP=AQ$。若 $AB=13$ cm、$AC=7$ cm 及 $AP=x$ cm，求 x。



(Figure 1) (图 1)

7. In figure 2, $BL=\frac{1}{3}BC$, $CM=\frac{1}{3}CA$ and $AN=\frac{1}{3}AB$. If the areas of ΔPQR and ΔABC are 6 cm^2 and $x\text{ cm}^2$ respectively, find x.

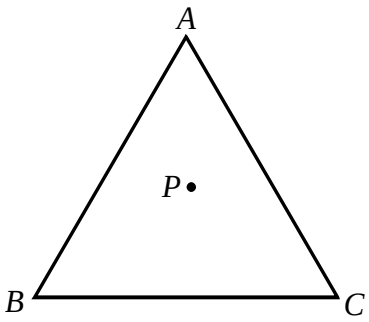
在图 2 中，$BL=\frac{1}{3}BC$、$CM=\frac{1}{3}CA$ 及 $AN=\frac{1}{3}AB$。若 ΔPQR 及 ΔABC 的面积依次为 6 cm^2 及 $x\text{ cm}^2$，求 x。



(Figure 2) (图 2)

8. ABC is an equilateral triangle of side $\sqrt{12}$ cm, and P is any point inside the triangle (as shown in figure 3). If the sum of the perpendicular distances from P to the three sides AB, BC and CA is x cm, find x.

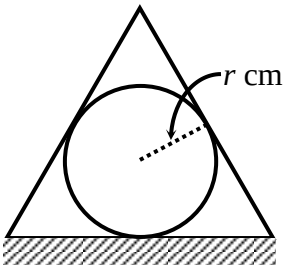
ABC 为一边长 $\sqrt{12}$ cm 的等边三角形，而 P 为此三角形内的任意一点 (如图 3 所示)。若 P 至三边 AB、BC 及 CA 的垂直距离的总和为 x cm，求 x。



(Figure 3) (图 3)

9. A sphere of radius r cm can just be covered on a table by a conical vessel of volume $\frac{8\pi r^2}{3}$ cm^3 (as shown in figure 4). Determine the largest possible value of r.

一半径为 r cm 的球体刚好被一体积为 $\frac{8\pi r^2}{3}$ cm^3 的圆锥形容器覆盖于桌上 (如图 4 所示)。求 r 的最大值。



(Figure 4) (图 4)

10. a, b, c, d are four integers. The arithmetic means of (i) a, b, c; (ii) b, c, d; (iii) a, b, d are respectively 13, 15 and 17. If the median of a, b, c and d is $c + 9$, find the largest value of c.

a、b、c、d 为四整数。已知 (i) a、b、c; (ii) b、c、d; 和 (iii) a、b、d 的算术平均数依次为 13、15 和 17。若 a、b、c 和 d 的中位数为 $c + 9$，求 c 的最大值。